

- (4)  $(a+b+1)(a+b+2) = [(a+b)+1][(a+b)+2] = (a+b)^2 + 3(a+b) + 2$   
 $= a^2 + 2ab + b^2 + 3a + 3b + 2$
- (5)  $(x+y-z)(x-y-z) = [(x-z)+y][(x-z)-y] = (x-z)^2 - y^2$   
 $= x^2 - 2xz + z^2 - y^2 = x^2 - y^2 + z^2 - 2xz$
- (6)  $(x^2+x+1)(x^2-x+1) = [(x^2+1)+x][(x^2+1)-x] = (x^2+1)^2 - x^2$   
 $= (x^4 + 2x^2 + 1) - x^2 = x^4 + x^2 + 1$

練習 1 5

- (1)  $(x^2+1)(x+1)(x-1) = (x^2+1)\{(x+1)(x-1)\} = (x^2+1)(x^2-1)$   
 $= (x^2)^2 - 1^2 = x^4 - 1$
- (2)  $(x^2+9y^2)(x+3y)(x-3y) = (x^2+9y^2)\{(x+3y)(x-3y)\} = (x^2+9y^2)(x^2-9y^2)$   
 $= (x^2)^2 - (9y^2)^2 = x^4 - 81y^4$
- (3)  $(x+3)^2(x-3)^2 = [(x+3)(x-3)]^2 = (x^2-9)^2 = (x^2)^2 - 2 \cdot x^2 \cdot 9 + 9^2$   
 $= x^4 - 18x^2 + 81$
- (4)  $(x+2y)^2(x-2y)^2 = [(x+2y)(x-2y)]^2 = (x^2-4y^2)^2$   
 $= (x^2)^2 - 2 \cdot x^2 \cdot 4y^2 + (4y^2)^2 = x^4 - 8x^2y^2 + 16y^4$

練習 1 6

- (1)  $3x^3 + 4x = x \cdot 3x^2 + x \cdot 4 = x(3x^2 + 4)$
- (2)  $x^2 + xy = x \cdot x + x \cdot y = x(x+y)$
- (3)  $2xy - 2y = 2y \cdot x - 2y \cdot 1 = 2y(x-1)$
- (4)  $8x^3 - 12x^2y = 4x^2 \cdot 2x - 4x^2 \cdot 3y = 4x^2(2x-3y)$
- (5)  $9abc + 3a^2c - 6ac^2 = 3ac \cdot 3b + 3ac \cdot a - 3ac \cdot 2c$   
 $= 3ac(3b + a - 2c) = 3ac(a + 3b - 2c)$

練習 1 7

- (1)  $(a+3)x + (a+3)y^2 = (a+3)(x+y^2)$
- (2)  $a(x-y) + 2(y-x) = a(x-y) - 2(x-y) = (a-2)(x-y)$

練習 1 8

- (1)  $x^2 + 4x + 4 = x^2 + 2 \cdot x \cdot 2 + 2^2 = (x+2)^2$
- (2)  $x^2 - 12x + 36 = x^2 - 2 \cdot x \cdot 6 + 6^2 = (x-6)^2$
- (3)  $x^2 + 8xy + 16y^2 = x^2 + 2 \cdot x \cdot 4y + (4y)^2 = (x+4y)^2$
- (4)  $9x^2 - 30xy + 25y^2 = (3x)^2 - 2 \cdot 3x \cdot 5y + (5y)^2 = (3x-5y)^2$

- (5)  $x^2 - 4y^2 = x^2 - (2y)^2 = (x+2y)(x-2y)$
- (6)  $25a^2 - 4 = (5a)^2 - 2^2 = (5a+2)(5a-2)$

練習 1 9

- (1)  $x^2 + 3x + 2 = (x+1)(x+2)$
- (2)  $x^2 + 7x + 10 = (x+2)(x+5)$
- (3)  $x^2 - x - 12 = (x+3)(x-4)$
- (4)  $x^2 + 4x - 12 = (x-2)(x+6)$
- (5)  $x^2 - 6x + 8 = (x-2)(x-4)$
- (6)  $x^2 - 8x + 15 = (x-3)(x-5)$

練習 2 0

- (1)  $x^2 - 9xy + 8y^2 = x^2 + (-y-8y)x + (-y) \cdot (-8y)$   
 $= (x-y)(x-8y)$
- (2)  $x^2 + xy - 20y^2 = x^2 + (-4y+5y)x + (-4y) \cdot 5y$   
 $= (x-4y)(x+5y)$

練習 2 1

- (1)  $3x^2 + 4x + 1 = (x+1)(3x+1)$
- (2)  $2x^2 + 7x + 3 = (x+3)(2x+1)$
- (3)  $2x^2 - 5x + 3 = (x-1)(2x-3)$
- (4)  $2x^2 + 3x - 2 = (x+2)(2x-1)$
- (5)  $3x^2 + 5x - 2 = (x+2)(3x-1)$
- (6)  $3x^2 - 7x - 6 = (x-3)(3x+2)$

$$\begin{array}{l} (7) \quad 3x^2 - 10xy + 3y^2 = (x-3y)(3x-y) \\ \begin{array}{r} 1 \quad \times \quad -3 \quad \rightarrow \quad -9 \\ 3 \quad \quad -1 \quad \rightarrow \quad -1 \\ \hline 3 \quad \quad 3 \quad \rightarrow \quad -10 \end{array} \\ (8) \quad 4x^2 + 3xy - 27y^2 = (x+3y)(4x-9y) \\ \begin{array}{r} 1 \quad \times \quad 3 \quad \rightarrow \quad 12 \\ 4 \quad \quad -9 \quad \rightarrow \quad -9 \\ \hline 4 \quad -27 \quad \rightarrow \quad 3 \end{array} \\ (9) \quad 6x^2 + ax - 15a^2 = (2x-3a)(3x+5a) \\ \begin{array}{r} 2 \quad \times \quad -3 \quad \rightarrow \quad -9 \\ 3 \quad \quad 5 \quad \rightarrow \quad 10 \\ \hline 6 \quad -15 \quad \rightarrow \quad 1 \end{array} \end{array}$$

練習 2 2

$$\begin{aligned} (1) \quad (x+y)^2 + 5(x+y) &= (x+y)\{(x+y)+5\} \\ &= (x+y)(x+y+5) \\ (2) \quad (x+y)^2 - 7(x+y) + 12 &= \{(x+y)-3\}\{(x+y)-4\} \\ &= (x+y-3)(x+y-4) \\ (3) \quad (x-y)^2 - 2(x-y) - 15 &= \{(x-y)+3\}\{(x-y)-5\} \\ &= (x-y+3)(x-y-5) \\ (4) \quad (x-3y)^2 - 4 &= (x-3y)^2 - 2^2 \\ &= \{(x-3y)+2\}\{(x-3y)-2\} \\ &= (x-3y+2)(x-3y-2) \end{aligned}$$

練習 2 3

$$\begin{aligned} (1) \quad x^2 + 6x + 9 - y^2 &= (x^2 + 6x + 9) - y^2 = (x+3)^2 - y^2 \\ &= \{(x+3)+y\}\{(x+3)-y\} \\ &= (x+y+3)(x-y+3) \\ (2) \quad x^2 - 2x + 1 - 4y^2 &= (x^2 - 2x + 1) - 4y^2 = (x-1)^2 - (2y)^2 \\ &= \{(x-1)+2y\}\{(x-1)-2y\} \\ &= (x+2y-1)(x-2y-1) \\ (3) \quad x^2 - y^2 + 2y - 1 &= x^2 - (y^2 - 2y + 1) = x^2 - (y-1)^2 \\ &= \{x+(y-1)\}\{x-(y-1)\} \\ &= (x+y-1)(x-y+1) \\ (4) \quad x^2 - 4y^2 + 4y - 1 &= x^2 - (4y^2 - 4y + 1) = x^2 - (2y-1)^2 \\ &= \{x+(2y-1)\}\{x-(2y-1)\} \\ &= (x+2y-1)(x-2y+1) \end{aligned}$$

練習 2 4

$$\begin{aligned} (1) \quad a^2 + ab + a + 2b - 2 &= (a+2)b + (a^2 + a - 2) \\ &= (a+2)b + (a+2)(a-1) \\ &= (a+2)\{b+(a-1)\} = (a+2)(a+b-1) \\ (2) \quad b^2 + ab + 4a - 16 &= (b+4)a + (b^2 - 16) \\ &= (b+4)a + (b+4)(b-4) \\ &= (b+4)\{a+(b-4)\} = (b+4)(a+b-4) \end{aligned}$$

練習 2 5

$$\begin{aligned} (1) \quad x^2 + (2y+1)x + (y+2)(y-1) &= \{x+(y-1)\}\{x+(y+2)\} \\ &= (x+y-1)(x+y+2) \\ (2) \quad x^2 + 3xy + 2y^2 + x - y - 6 &= x^2 + (3y+1)x + (2y^2 - y - 6) \\ &= x^2 + (3y+1)x + (y-2)(2y+3) \\ &= \{x+(y-2)\}\{x+(2y+3)\} \\ &= (x+y-2)(x+2y+3) \\ (3) \quad 2x^2 + 3xy + y^2 - 9x - 7y + 10 &= 2x^2 + (3y-9)x + (y^2 - 7y + 10) \\ &= 2x^2 + (3y-9)x + (y-2)(y-5) \\ &= \{x+(y-2)\}\{2x+(y-5)\} \\ &= (x+y-2)(2x+y-5) \end{aligned}$$

$$\begin{array}{r} 1 \quad \times \quad y-1 \quad \rightarrow \quad y-1 \\ 1 \quad \quad y+2 \quad \rightarrow \quad y+2 \\ \hline 1 \quad (y+2)(y-1) \quad 2y+1 \end{array}$$

$$\begin{array}{r} 1 \quad \times \quad y-2 \quad \rightarrow \quad y-2 \\ 1 \quad \quad 2y+3 \quad \rightarrow \quad 2y+3 \\ \hline 1 \quad (y-2)(2y+3) \quad 3y+1 \end{array}$$

$$\begin{array}{r} 1 \quad \times \quad y-2 \quad \rightarrow \quad 2y-4 \\ 2 \quad \quad y-5 \quad \rightarrow \quad y-5 \\ \hline 2 \quad (y-2)(y-5) \quad 3y-9 \end{array}$$

練習問題 1

- (1)  $A+B=(x^3-2x^2+3x+6)+(x^3+4x^2+2)$   
 $=x^3-2x^2+3x+6+x^3+4x^2+2$   
 $=(x^3+x^3)+(-2x^2+4x^2)+3x+(6+2)$   
 $=2x^3+2x^2+3x+8$
- (2)  $3A+B=3(x^3-2x^2+3x+6)+(x^3+4x^2+2)$   
 $=3x^3-6x^2+9x+18+x^3+4x^2+2$   
 $=(3x^3+x^3)+(-6x^2+4x^2)+9x+(18+2)$   
 $=4x^3-2x^2+9x+20$
- (3)  $2A-3B=2(x^3-2x^2+3x+6)-3(x^3+4x^2+2)$   
 $=2x^3-4x^2+6x+12-3x^3-12x^2-6$   
 $=(2-3)x^3+(-4-12)x^2+6x+(12-6)$   
 $=-x^3-16x^2+6x+6$
- (4)  $2a^5 \times 3a^4 = (2 \times 3) \times a^{5+4} = 6a^9$
- (5)  $3x^2y \times (-2x)^3 = 3x^2y \times (-2)^3 \times x^3 = [3 \times (-2)^3] \times x^{2+3} \times y = -24x^5y$
- (6)  $-(3x^2y)^4 = -[3^4 \times (x^2)^4 \times y^4] = -(81 \times x^{2 \times 4} \times y^4) = -81x^8y^4$

練習問題 2

練習問題 3

- (1)  $-3xy(2x-3y) = -3xy \cdot 2x + 3xy \cdot 3y = -6x^2y + 9xy^2$
- (2)  $(x-3)(x^2+2x-5) = x(x^2+2x-5) - 3(x^2+2x-5)$   
 $= x^3 + 2x^2 - 5x - 3x^2 - 6x + 15$   
 $= x^3 + (2x^2 - 3x^2) + (-5x - 6x) + 15$   
 $= x^3 - x^2 - 11x + 15$
- (3)  $(2x+3y)^2 = (2x)^2 + 2 \cdot 2x \cdot 3y + (3y)^2 = 4x^2 + 12xy + 9y^2$
- (4)  $(x-2y)^2 = x^2 - 2 \cdot x \cdot 2y + (2y)^2 = x^2 - 4xy + 4y^2$
- (5)  $(x-7)(x+7) = x^2 - 7^2 = x^2 - 49$
- (6)  $(5a+4b)(5a-4b) = (5a)^2 - (4b)^2 = 25a^2 - 16b^2$
- (7)  $(4x-3)(x-2) = 4 \cdot 1x^2 + [4 \cdot (-2) + (-3) \cdot 1]x + (-3) \cdot (-2)$   
 $= 4x^2 - 11x + 6$
- (8)  $(x+5y)(3x-7y) = 1 \cdot 3x^2 + [1 \cdot (-7) + 5 \cdot 3]xy + 5 \cdot (-7)y^2$   
 $= 3x^2 + 8xy - 35y^2$

練習問題 4

- (1)  $12x^3y - 4x^2y^2 = 4x^2y \cdot 3x - 4x^2y \cdot y = 4x^2y(3x-y)$
- (2)  $x^2 + 10x + 25 = x^2 + 2 \cdot x \cdot 5 + 5^2 = (x+5)^2$
- (3)  $4x^2 - 20x + 25 = (2x)^2 - 2 \cdot 2x \cdot 5 + 5^2 = (2x-5)^2$
- (4)  $9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x+4y)(3x-4y)$
- (5)  $x^2 + 4x - 5 = (x-1)(x+5)$
- (6)  $x^2 - 2xy - 8y^2 = (x+2y)(x-4y)$
- (7)  $3x^2 - 8x + 4 = (x-2)(3x-2)$
- (8)  $2x^2 + 9xy - 5y^2 = (x+5y)(2x-y)$

練習問題 5

- (1)  $(x-y-1)(x+y-1) = (x-1)-y)((x-1)+y)$   
 $= (x-1)^2 - y^2 = (x^2 - 2x + 1) - y^2$   
 $= x^2 - y^2 - 2x + 1$
- (2)  $(x+y-z)(x-y+z) = (x+(y-z))(x-(y-z))$   
 $= x^2 - (y-z)^2 = x^2 - (y^2 - 2yz + z^2)$   
 $= x^2 - y^2 - z^2 + 2yz$
- (3)  $(4x^2+9)(2x+3)(2x-3) = (4x^2+9)((2x+3)(2x-3))$   
 $= (4x^2+9)(4x^2-9) = (4x^2)^2 - 9^2$   
 $= 16x^4 - 81$
- (4)  $(3a+2)^2(3a-2)^2 = ((3a+2)(3a-2))^2 = (9a^2-4)^2$   
 $= (9a^2)^2 - 2 \cdot 9a^2 \cdot 4 + 4^2$   
 $= 81a^4 - 72a^2 + 16$

練習問題 6

- (1)  $(x+1)^2 - 7(x+1) - 30 = ((x+1)+3)((x+1)-10)$   
 $= (x+4)(x-9)$
- (2)  $x^2 - 12x + 36 - y^2 = (x^2 - 12x + 36) - y^2$   
 $= (x-6)^2 - y^2$   
 $= ((x-6)+y)((x-6)-y)$   
 $= (x+y-6)(x-y-6)$

$$\begin{aligned} (3) \quad & a^2 + ab - 3a + 2b - 10 = (a+2)b + a^2 - 3a - 10 \\ & = (a+2)b + (a+2)(a-5) \\ & = (a+2)\{b+(a-5)\} \\ & = (a+2)(a+b-5) \end{aligned}$$

$$\begin{aligned} (4) \quad & 3x^2 - 2xy - y^2 + 5x + 3y - 2 \\ & = 3x^2 + (-2y+5)x - (y^2 - 3y + 2) \\ & = 3x^2 + (-2y+5)x - (y-1)(y-2) \\ & = \{x-(y-2)\}(3x+(y-1)) \\ & = (x-y+2)(3x+y-1) \end{aligned}$$

$$\frac{1}{3} \begin{array}{r} \times \\ \hline \begin{array}{l} -(y-2) \longrightarrow -3y+6 \\ y-1 \longrightarrow y-1 \\ \hline -(y-1)(y-2) \longrightarrow -2y+5 \end{array} \end{array}$$

発展 3 次式の展開と因数分解

練習 1

$$\begin{aligned} (1) \quad & (x+2)^3 = x^3 + 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 2^2 + 2^3 \\ & = x^3 + 6x^2 + 12x + 8 \\ (2) \quad & (x-1)^3 = x^3 - 3 \cdot x^2 \cdot 1 + 3 \cdot x \cdot 1^2 - 1^3 \\ & = x^3 - 3x^2 + 3x - 1 \\ (3) \quad & (x+3y)^3 = x^3 + 3 \cdot x^2 \cdot 3y + 3 \cdot x \cdot (3y)^2 + (3y)^3 \\ & = x^3 + 9x^2y + 27xy^2 + 27y^3 \end{aligned}$$

練習 2

$$\begin{aligned} (1) \quad & (x+2)(x^2-2x+4) = (x+2)(x^2-x \cdot 2+2^2) \\ & = x^3+2^3=x^3+8 \\ (2) \quad & (x-1)(x^2+x+1) = (x-1)(x^2+x \cdot 1+1^2) \\ & = x^3-1^3=x^3-1 \\ (3) \quad & (2x+y)(4x^2-2xy+y^2) = (2x+y)\{(2x)^2-2x \cdot y+y^2\} \\ & = (2x)^3+y^3=8x^3+y^3 \\ (4) \quad & (2x-3y)(4x^2+6xy+9y^2) = (2x-3y)\{(2x)^2+2x \cdot 3y+(3y)^2\} \\ & = (2x)^3-(3y)^3=8x^3-27y^3 \end{aligned}$$

練習 3

$$\begin{aligned} (1) \quad & x^3+8=x^3+2^3=(x+2)(x^2-x \cdot 2+2^2) \\ & = (x+2)(x^2-2x+4) \\ (2) \quad & x^3+27=x^3+3^3=(x+3)(x^2-x \cdot 3+3^2) \\ & = (x+3)(x^2-3x+9) \\ (3) \quad & x^3-64=x^3-4^3=(x-4)(x^2+x \cdot 4+4^2) \\ & = (x-4)(x^2+4x+16) \\ (4) \quad & 27x^3-8y^3=(3x)^3-(2y)^3=(3x-2y)\{(3x)^2+3x \cdot 2y+(2y)^2\} \\ & = (3x-2y)(9x^2+6xy+4y^2) \end{aligned}$$